EXERCISE – I   SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. The area of the triangle formed by the positive x–axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

(A) $3\sqrt{3}$ sq. units (B) $2\sqrt{3}$ sq. units
(C) $4\sqrt{3}$ sq. units (D) $\sqrt{3}$ sq. units

2. Equation of the normal to the curve $y = -\sqrt{x} + 2$ at the point of its intersection with the curve $y = \tan(\tan^{-1} x)$ is

(A) $2x - y - 1 = 0$ (B) $2x - y + 1 = 0$
(C) $2x + y - 3 = 0$ (D) None of these

3. The abscissa of the point on the curve $ay^2 = x^3$, the normal at which cuts off equal intercepts from the coordinate axes is

(A) $\frac{2a}{9}$ (B) $\frac{4a}{9}$ (C) $-\frac{4a}{9}$ (D) $-\frac{2a}{9}$

4. If the tangent to the curve $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ at $\theta = \frac{\pi}{3}$ makes an angle α $(0 \le \alpha < \pi)$ with x–axis, then α equals

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

5. The x–intercept of the tangent at any arbitrary point of the curve $\frac{a}{x^2} + \frac{b}{y^2} = 1$ is proportional to

(A) square of the abscissa of the point of tangency
(B) square root of the abscissa of the point of tangency
(C) cube of the abscissa of the point of tangency
(D) cube root of the abscissa of the point of tangency.

6. If curve $y = 1 - ax^2$ and $y = x^2$ intersect orthogonally then the value of a is

(A) 1/2 (B) 1/3 (C) 2 (D) 3

7. The coordinates of the point of the parabola $y^2 = 8x$, which is at minimum distance from the circle $x^2+(y+6)^2=1$ are

(A) (2, – 4) (B) (18, –12)
(C) (2, 4) (D) None of these

8. The length of the subtangent to the curve $\sqrt{x} + \sqrt{y} = 3$ at the point (4, 1) is

(A) 2 (B) 1/2 (C) 3 (D) 4

9. For a curve $\frac{(\text{length of normal})^2}{(\text{length of tangent})^2}$ is equal to

(A) (subnormal) / (subtangent)
(B) (subtangent) / (subnormal)
(C) (subnormal) / $(\text{subtangent})^2$ (D) None of these

10. Water is poured into an inverted conical vessel of which the radius of the base is 2m and height 4m, at the rate of 77 litre/minute. The rate at which the water level is rising at the instant when the depth is 70 cm is: (use $\pi = 22/7$)

(A) 10 cm/min (B) 20 cm/min
(C) 40 cm/min (D) None of these

11. If the tangent at each point of the curve $y = \frac{2}{3}x^3 - 2ax^2 + 2x + 5$ makes an acute angle with the positive direction of x–axis, then

(A) $a \ge 1$ (B) $-1 \le a \le 1$
(C) $a \le -1$ (D) None of these

12. The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point

(A) (–a, be) (B) $\left(-a, \frac{b}{e}\right)$ (C) $\left(a, \frac{b}{e}\right)$ (D) (0, b)

13. All points on the curve $y^2 = 4a\left(x + a\sin\frac{x}{a}\right)$ at which the tangents are parallel to the axis of x, lie on a

(A) circle (B) parabola (C) line (D) None of these

14. A curve is represented by the equations, $x = \sec^2 t$ and $y = \cot t$ where t is a parameter. If the tangent at the point P on the curve where $t = \pi/4$ meets the curve again at the point Q then |PQ| is equal to

(A) $\frac{5\sqrt{3}}{2}$ (B) $\frac{5\sqrt{5}}{2}$ (C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{3\sqrt{5}}{2}$

15. If the subnormal at any point on $y = a^{1-n} x^n$ is of constant length, then the value of n is
(A) 1 (B) 1/2 (C) 2 (D) –2

16. The curves $x^3 + p\,xy^2 = -2$ and $3x^2y - y^3 = 2$ are orthogonal for
(A) p = 3 (B) p = –3 (C) no value of p (D) p = ± 3

17. If curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ intersect orthogonally, then
(A) a + b = 0 (B) $a^2 = b^2$
(C) a + b = c (D) None of these

18. The ordinate of $y = (a/2)(e^{x/a} + e^{-x/a})$ is the geometric mean of the length of the normal and the quantity
(A) a/2 (B) a (C) e (D) None of these

19. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is
(A) $\pi/2$ (B) $\pi/6$ (C) $\pi/4$ (D) $\pi/3$

20. If the tangent at P of the curve $y^2 = x^3$ intersects the curve again at Q and the straight lines OP, OQ make angles α, β with the x–axis, where 'O' is the origin, then $\tan\alpha/\tan\beta$ has the value equal to
(A) –1 (B) –2 (C) 2 (D) $\sqrt{2}$

21. Water is being poured on to a cylindrical vessel at the rate of 1 m^3/min. If the vessel has a circular base of radius 3m, the rate at which the level of water is rising in the vessel is
(A) 1/9 π m/min (B) 0 π m/min
(C) 1/3 π m/min (D) 3 π m/min

22. Find the number of points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.
(A) 1 (B) 2 (C) 3 (D) None of these

23. If at any point on a curve the subtangent and subnormal are equal, then the tangent is equal to
(A) ordinate (B) $\sqrt{2}$ ordinate
(C) $\sqrt{2(\text{ordinate})}$ (D) None of these

24. The length of the normal to the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, at $\theta = \frac{\pi}{2}$ is
(A) 2a (B) $a\sqrt{2}$ (C) a/2 (D) $a/\sqrt{2}$

25. The number of values of c such that the straight line 3x + 4y = c touches the curve $\frac{x^4}{2} = x + y$ is
(A) 0 (B) 1 (C) 2 (D) 4

26. The beds of two rivers (within a certain region) are a parabola $y = x^2$ and a straight line y = x – 2. These rivers are to be connected by a straight canal. The co-ordinates of the ends of the shortest canal can be
(A) $\left(\frac{1}{2},\frac{1}{4}\right)$ and $\left(-\frac{11}{8},\frac{5}{8}\right)$ (B) $\left(\frac{1}{2},\frac{1}{4}\right)$ and $\left(\frac{11}{8},-\frac{5}{8}\right)$
(C) (0, 0) and (1, –1) (D) None of these

27. The points(s) of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is intersected by the curve xy = 1 – y is/are given by
(A) (0, –1) (B) (0, 1)
(C) (1, 1) (D) None of these

28. If the area of the triangle included between the axes and any tangent to the curve $x^n y = a^n$ is constant, then n is equal to
(A) 1 (B) 2 (C) $\frac{3}{2}$ (D) $\frac{1}{2}$

29. At (0, 0), the curve $y^2 = x^3 + x^2$
(A) touches X-axis
(B) bisects the angle between the axes
(C) makes an angle of 60° with OX
(D) None of these

30. For the curve $x = t^2 - 1$, $y = t^2 - t$, the tangent line is perpendicular to x-axis where
(A) t = 0 (B) $t = \infty$ (C) $t = \frac{1}{\sqrt{3}}$ (D) $t = -\frac{1}{\sqrt{3}}$